Production of genuine entangled states of four atomic qubits

Gui-Yun Liu and Le-Man Kuang*†

Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, and Department of Physics, Hunan Normal University, Changsha 410081, China

We propose an optical scheme to generate genuine entangled states of four atomic qubits in optical cavities using a single-photon source, beam splitters and single photon detectors. We show how to generate deterministically sixteen orthonormal and independent genuine entangled states of four atomic qubits. It is found that the sixteen genuine entangled states form a new type of representation of the four-atomic-qubit system, i.e., the genuine entangled-state representation. This representation brings new interesting insight onto better understanding multipartite entanglement.

PACS numbers: 03.67.Mn, 03.65.Ud

I. INTRODUCTION

Quantum entanglement plays an important role in quantum information processing and quantum mechanics. While bipartite entanglement is well understood, multiparty entanglement is still under intensive research. Multipartite entanglement is thus not a straightforward extension of bipartite entanglement and gives rise to new phenomena which can be exploited in quantum information and quantum computing processes. For example, there are quantum communication protocols that require multiparty entanglement such as universal error correction [1], quantum secret sharing [2], and telecloning [3]. Also, highly entangled multipartite states are needed for one-way quantum computing [4]. In fact, all known quantum algorithms do work with multipartite entanglement. On the other hand, multipartite entangled states provide a stronger test of local realism. As a general rule, one can say that the more particles are entangled, the more clearly nonclassical effects are exhibited, and the more useful the states are for quantum applications. In addition, multipartite entanglement is expected to play a key role on quantum-phase transition phenomena [5].

Although multipartite entanglement is ubiquitous in many-body quantum systems, it is very difficult both to characterize and to quantify it. Up to now, there is no unique way to define multipartite entanglement, even in the simplest case of pure states. The presence of multipartite entanglement clearly depends on the partitioning that one imposes in order to group the individual subsystems into parties. Furthermore, given a fixed partition, one can single out a hierarchy of different levels of multipartite entanglement which establishes a smooth connection between the two limiting cases of a fully separable state, where the parties are all disentangled, and of fully inseparable states, where entanglement exists across any global bisection, and the parties are supposed to share genuine multipartite entanglement. Genuine multipartite entanglement is distinguished from other types of entanglement by the participation of all parties in quantum correlations, and it is particularly distinct from biseparable entanglement. Yeo and Chua [6] indicated that an arbitrary two-qubit state can be faithfully teleported by the use of a genuine four-qubit entangled state. And it was found that a genuine $2N$-qubit entangled state can be used to teleport an arbitrary N-qubit state [7], and a genuine $(2N+1)$-qubit entangled state can realize controlled teleportation of an arbitrary N-qubit state [8]. One of the important issues regarding many-body quantum systems is to generate and to verify genuine multipartite entanglement among parties. The purpose of this paper is to propose an optical scheme to produce genuine entangled states (GESs) of four atomic qubits in optical cavities. This paper is organized as follows. In section 2, we propose our theoretical model. In section 3, we show how to create sixteen orthonormal and independent genuine entangled states of four atomic qubits. We shall conclude our paper with discussions and remarks in the last section.

II. THEORETICAL MODEL

The basic setup for genuine entanglement generation of four atomic qubits is indicated in Fig. 1 where we make use a Mach-Zehnder (MZ) interferometer consisting of two 50/50 optical beam splitters (BSs). The input light field is bifurcated at the first BS, guided to interact sequentially with the atomic qubits which are placed in four separate high-finesse optical cavities in which four atomic qubit are placed, and then recombined at the second BS. We consider atomic qubits based on degenerate hyperfine states denoted by $|0\rangle_i$ and $|1\rangle_i$ $(i=1,2,3,4)$. A single pulse of light is passed through an optical interferometer, with the different arms of the interferometer corresponding to different photon polarization states. Each polarization state interacts with a different internal atomic state through the mechanism proposed in Ref. [9]. Under the condition of the large detuning, the atom-photon evolution can be described by the following uni-

*Author to whom any correspondence should be addressed.
†Email: lmkuang@hunnu.edu.cn

tary transformation [10]

$$\hat{U}_i = \exp[-i\phi(\hat{a}_U^\dagger \hat{a}_U|0\rangle_i\langle 0| + \hat{a}_L^\dagger \hat{a}_L|1\rangle_i\langle 1|)], \quad (1)$$

where $\hat{a}_{U,L}^\dagger$ and $\hat{a}_{U,L}$ are the creation and annihilation operators of the optical fields corresponding to the upper arm and the low arm, respectively. $|0\rangle_i$ and $|1\rangle_i$ are the atomic degenerate hyperfine states corresponding to the four atomic qubits ($i = 1, 2, 3, 4$). The interaction is governed by the phase-shift $\phi = \frac{d^2\mathcal{E}^2\tau}{\hbar^2\Delta}$, where d is the electrical dipole moment and $\mathcal{E} = \sqrt{\hbar\omega/2\epsilon_0 V}$ is the electrical field of a photon for laser with the frequency ω and the mode volume V, τ is the atom-photon interaction time, Δ is the detuning between the atomic resonance and laser frequencies. The unitary evolution of the whole system indicated in Fig. 1 can be described by the following unitary transformation

$$\hat{U} = \hat{U}_{BS2}\hat{U}_4\hat{U}_3\hat{U}_2\hat{U}_1\hat{U}_{BS1}, \quad (2)$$

where $\hat{U}_{BSi}$ $(i = 1, 2)$is the 50/50 BS operator given by

$$\hat{U}_{BSi} = \exp\left[-i\frac{\pi}{4}(\hat{a}_U^\dagger \hat{a}_L + \hat{a}_L^\dagger \hat{a}_U)\right]. \quad (3)$$

In the following we will show that genuine entangled states of four atomic qubits by using above unitary transformation and single photon detections.

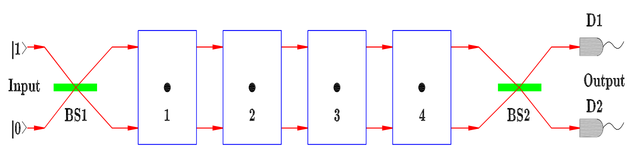

FIG. 1: Schematic setup to generate genuine four atomic qubits. The input single-photon field is bifurcated at the first BS, guided to interact sequentially with the atomic qubits placed in four separate high-finesse optical cavities, and then recombined at the second BS. Each cavity has an atomic qubit. D1 and D2 are two single photon detectors.

III. GENUINE ENTANGLED STATES OF FOUR ATOMIC QUBITS

In this section, we show how to prepare genuine entangled states of four atomic qubits using the setup indicated in Fig. 1. Let us consider the case of the single-photon input. In this case, the upper channel is in the single-photon state $|1\rangle$ while the lower channel is in the vacuum state $|0\rangle$. Then the initial state of the optical field is $|\Psi_i\rangle_O = |10\rangle$ while the initial state of the four atomic qubits can be supposed as

$$|\Psi_i\rangle_A = |\Phi_1\rangle \otimes |\Phi_2\rangle \otimes |\Phi_3\rangle \otimes |\Phi_4\rangle, \quad (4)$$

where the initial state of the single atomic qubit is given by

$$|\Phi_i\rangle = \cos\theta_i|0\rangle_i + \sin\theta_i|1\rangle_i, \quad (i = 1, 2, 3, 4), \quad (5)$$

which implies that the initial state of the whole system is $|\Psi_i\rangle = |\Psi_i\rangle_O \otimes |\Psi_i\rangle_A$.

The state of the system at the output of the MZ interferometer is then given by $|\Psi_f\rangle = \hat{U}|\Psi_i\rangle$ where the unitary transformation $\hat{U}$ is given by Eq. (3). It is easy to find that the output state $|\Psi_f\rangle$ has the following expression

$$|\Psi_f\rangle = |01\rangle \otimes |\chi^{'}(\phi)\rangle + |10\rangle \otimes |\chi^{''}(\phi)\rangle, \quad (6)$$

where we have got rid of a global phase factor, $|01\rangle$ and $|10\rangle$ are quantum states of two light fields, $|\chi^{''}(\phi)\rangle$ and $|\chi^{''}(\phi)\rangle$ are quantum states of the four atomic qubits given by

$$\begin{aligned} \left|\chi^{'}(\phi)\right\rangle &= \cos 2\phi|A_+\rangle + \cos\phi(|B\rangle + |C\rangle) + |D\rangle, & (7) \\ \left|\chi^{''}(\phi)\right\rangle &= \sin 2\phi A_-\rangle + \sin\phi(|B\rangle - |C\rangle), & (8) \end{aligned}$$

which are generally entangled states. Here we have introduced

$$\begin{aligned} |A_\pm\rangle &= c_1c_2c_3c_4\,|0000\rangle \pm s_1s_2s_3s_4\,|1111\rangle, & (9) \\ |B\rangle &= c_1c_2s_3c_4\,|0010\rangle + c_1s_2c_3c_4\,|0100\rangle \\ &\quad + s_1c_2c_3c_4\,|1000\rangle + c_1c_2c_3s_4\,|0001\rangle, & (10) \\ |C\rangle &= s_1s_2s_3c_4\,|1110\rangle + c_1s_2s_3s_4\,|0111\rangle \\ &\quad + s_1s_2c_3s_4\,|1101\rangle + s_1c_2s_3s_4\,|1011\rangle], & (11) \\ |D\rangle &= c_1s_2s_3c_4\,|0110\rangle + s_1s_2c_3c_4\,|1100\rangle \\ &\quad + s_1c_2s_3c_4\,|1010\rangle + c_1c_2s_3s_4\,|0011\rangle \\ &\quad + c_1s_2c_3s_4\,|0101\rangle_a + s_1c_2c_3s_4\,|1001\rangle, & (12) \end{aligned}$$

where we have used notations $c_i = \cos\theta_i$ and $s_i = \sin\theta_i$.

Eq. (6) indicates that entangled states of the four atomic qubits can be produced through making quantum measurements upon quantum states of the output light and controlling the phase ϕ, i.e., the photon-atom interaction time τ. In fact, when $\phi = \pi/2$, if the photodetector D2 is triggered, at the same time the photodetector D1 detects the null result, the atomic-qubit state will collapse onto the following superposition state

$$|\chi^{'}(\pi/2)\rangle = \frac{1}{\sqrt{\Gamma}_1}(|\Lambda_1^{'}\rangle + |\Lambda_2^{'}\rangle), \quad (13)$$

where $|\Lambda_1^{'}\rangle$ and $|\Lambda_2^{'}\rangle$ are defined by

$$\begin{aligned} |\Lambda_1^{'}\rangle &= c_1c_2c_3c_4|0000\rangle - c_1s_2s_3c_4|0110\rangle \\ &\quad - c_1c_2s_3s_4|0011\rangle - c_1s_2c_3s_4|0101\rangle, & (14) \\ |\Lambda_2^{'}\rangle &= s_1s_2s_3s_4|1111\rangle - s_1s_2c_3c_4|1100\rangle \\ &\quad - s_1c_2s_3c_4|1010\rangle - s_1c_2c_3s_4|1001\rangle), & (15) \end{aligned}$$

where the normalization constant is given by

$$\Gamma_1 = \frac{1}{2}\left(1+\cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4\right). \quad (16)$$

Similarly, when $\phi = \pi/2$, if the photodetector D1 is triggered, and the photodetector D2 detects the null result, the atomic qubits state will become the following superposition state

$$|\chi^{''}(\pi/2)\rangle = \frac{1}{\sqrt{\Gamma_1}}(|\Lambda_1^{''}\rangle - |\Lambda_2^{''}\rangle) \quad (17)$$

where $|\Lambda_1^{''}\rangle$ and $|\Lambda_2^{''}\rangle$ are defined by

$$\begin{aligned} |\Lambda_1^{''}\rangle &= s_1 s_2 s_3 c_4 |1110\rangle + c_1 s_2 s_3 s_4 |0111\rangle \\ &\quad + s_1 s_2 c_3 s_4 |1101\rangle + s_1 c_2 s_3 s_4 |1011\rangle, \quad (18) \\ |\Lambda_2^{''}\rangle &= c_1 c_2 s_3 c_4 |0010\rangle + c_1 s_2 c_3 c_4 |0100\rangle \\ &\quad + s_1 c_2 c_3 c_4 |1000\rangle + c_1 c_2 c_3 s_4 |0001\rangle), \quad (19) \end{aligned}$$

where the normalization constant is given by

$$\Gamma_2 = \frac{1}{2}\left(1-\cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4\right). \quad (20)$$

It is obvious to see that quantum states of the four atomic qubits $|\chi^{'}(\pi/2)\rangle$ and $|\chi^{''}(\pi/2)\rangle$ given by Eqs. (13) and (17) are generally entangled states. In what follows we show that under certain conditions, $|\chi^{'}(\pi/2)\rangle$ and $|\chi^{''}(\pi/2)\rangle$ will be GESs of four atomic qubits. In order to observe this, we now investigate entanglement properties of entangled states $|\chi^{'}(\pi/2)\rangle$ and $|\chi^{''}(\pi/2)\rangle$ in which quantum state of any two atomic qubits is a mixed state. The degree of entanglement of a two-qubit mixed state can be measured in terms of the concurrence function defined in Ref. [11]. It is straightforward to find the concurrence between any two atomic qubits for quantum states $|\chi^{'}(\pi/2)\rangle$ and $|\chi^{''}(\pi/2)\rangle$ to be

$$C(|\chi^{'}\rangle) = \max(0, \lambda_+), C(|\chi^{''}\rangle) = \max(0, \lambda_-). \quad (21)$$

where the two parameters $\lambda_{\pm}$ are given by

$$\lambda_{\pm} = \frac{|\cos 2\theta_1 \cos 2\theta_2 \sin 2\theta_3 \sin 2\theta_4|}{1 \pm \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4}. \quad (22)$$

The amount of entanglement between any two atom-qubit pairs in four-atom-qubit entangled states $|\chi^{'}(\pi/2)\rangle$ and $|\chi^{''}(\pi/2)\rangle$ can be measured in terms of the von Neumann entropy [6]. We find that the von Neumann entropy for any two qubit-pairs in four-atom-qubit entangled states $|\chi^{'}(\pi/2)\rangle$ and $|\chi^{''}(\pi/2)\rangle$ is given by

$$\begin{aligned} S(|\chi^{'}\rangle) &= 1 - \frac{1}{2}[(1+\delta_+)\log_2(1+\delta_+) \\ &\quad + (1-\delta_+)\log_2(1-\delta_+)], \quad (23) \\ S(|\chi^{''}\rangle) &= 1 - \frac{1}{2}[(1+\delta_-)\log_2(1+\delta_-) \\ &\quad + (1-\delta_-)\log_2(1-\delta_-)], \quad (24) \end{aligned}$$

where we have introduced

$$\delta_{\pm} = \frac{\cos 2\theta_3 \cos 2\theta_4 \pm \cos 2\theta_1 \cos 2\theta_2}{1 \pm \cos 2\theta_1 \cos 2\theta_2 \cos 2\theta_3 \cos 2\theta_4}. \quad (25)$$

A further calculation indicates that the amount of entanglement between any one atomic qubit and the other three atomic qubits in four-atom-qubit entangled states $|\chi^{'}(\pi/2)\rangle$ and $|\chi^{''}(\pi/2)\rangle$ is the same as that of any two atomic-qubit pairs.

In particular, from Eqs. (13) and (17) we can see that when $\theta_1 = \theta_1 = \theta_2 = \theta_3 = \theta_4 = \pm\pi/4$, the resultant entangled states become

$$\begin{aligned} |\overline{\chi^{'}}\rangle &= \frac{1}{2\sqrt{2}}(|0000\rangle + |1111\rangle - |0110\rangle - |1100\rangle \\ &\quad - |1010\rangle - |0011\rangle - |0101\rangle - |1001\rangle), \quad (26) \\ |\overline{\chi^{''}}\rangle &= \frac{1}{2\sqrt{2}}(|1110\rangle + |0111\rangle + |1101\rangle + |1011\rangle \\ &\quad - |0010\rangle - |0100\rangle - |1000\rangle - |0001\rangle). \quad (27) \end{aligned}$$

For above entangled states, making Eqs. (21-25) we find that $C(|\chi^{'}\rangle) = C(|\chi^{''}\rangle) = 0$ and $S(|\chi^{'}\rangle) = S(|\chi^{''}\rangle) = 1$. This implies that there is absolutely zero entanglement between nay one atomic qubit and any other atomic qubit, and the entanglement is purely between pairs of atomic qubits. Therefore, these entangled states given by Eqs. (26) and (27) are GESs for four atomic qubits.

We then have a look at the probability of success to get above GESs. From Eq. (6), it is straightforward to see that when the second photodetector D2 clicks, at the same time the photodetector D1 detects the null result, the atomic-qubit state will collapse onto the GES $|\overline{\chi^{'}}\rangle$ with the probability of success being 1/2, and when the first photodetector D1 clicks, at the same time the photodetector D2 detects the null result, the atomic-qubit state will collapse onto the GES $|\overline{\chi^{''}}\rangle$ with the probability of success being 1/2 too. It is interesting to note that the GES $|\overline{\chi^{''}}\rangle$ (or $|\overline{\chi^{'}}\rangle$) can be obtained by applying the Pauli operator $\hat{\sigma}_4^y$ to fourth atomic qubit of the GES $|\overline{\chi^{''}}\rangle$ (or $|\overline{\chi^{'}}\rangle$), i.e.,

$$\hat{\sigma}_4^y|\overline{\chi^{'}}\rangle = |\overline{\chi^{''}}\rangle, \quad \hat{\sigma}_4^y|\overline{\chi^{''}}\rangle = |\overline{\chi^{'}}\rangle, \quad (28)$$

which implies that the GESs $|\overline{\chi^{'}}\rangle$ and $|\overline{\chi^{''}}\rangle$ can be produced with the probability of success being one through making use of $\hat{\sigma}^y$ operation on fourth atomic qubit. Hence, in our scheme the GESs $|\overline{\chi^{'}}\rangle$ and $|\overline{\chi^{''}}\rangle$ can be produced deterministically through making single photon detections of output fields and unitary transformation ($\hat{\sigma}^y$) upon the fourth atomic qubit.

It is interesting to see that starting with the GES $|\overline{\chi^{'}}\rangle$ or $|\overline{\chi^{''}}\rangle$ we can generate a basis of sixteen orthonormal states by applying Pauli operators of atomic qubits to the GES $|\overline{\chi^{'}}\rangle$ or $|\overline{\chi^{''}}\rangle$. For instance, for the GES $|\overline{\chi^{'}}\rangle$ we can obtain the following sixteen GESs

$$\begin{aligned} |\varphi_1\rangle_\mu &= \sigma_1^0 \sigma_2^\mu |\overline{\chi^{'}}\rangle, \quad |\varphi_2\rangle_\mu = \sigma_1^3 \sigma_2^\mu |\overline{\chi^{'}}\rangle, \quad (29) \\ |\varphi_3\rangle_\mu &= \sigma_1^0 \sigma_2^\mu \sigma_3^3 |\overline{\chi^{'}}\rangle, \quad |\varphi_4\rangle_\mu = \sigma_1^3 \sigma_2^\mu \sigma_3^3 |\overline{\chi^{'}}\rangle, \quad (30) \end{aligned}$$

where σ_i^{μ} ($\mu = 0, 1, 2, 3$) denotes the μ-th component of the Pauli operator for the i-th atomic qubit with σ_i^0 being the unit matrix. The above sixteen GESs can be explicitly expressed as

$$\begin{aligned}
|\varphi_1\rangle_0 &= \frac{1}{\sqrt{8}}(|0000\rangle + |1111\rangle - |0110\rangle - |1100\rangle \\
&\quad - |1010\rangle - |0011\rangle - |0101\rangle - |1001\rangle), \quad (31)\\
|\varphi_1\rangle_1 &= \frac{1}{\sqrt{8}}(-|1110\rangle - |0111\rangle - |1101\rangle + |1011\rangle \\
&\quad - |0010\rangle + |0100\rangle - |1000\rangle - |0001\rangle), \quad (32)\\
|\varphi_1\rangle_2 &= \frac{1}{\sqrt{8}}(|1110\rangle + |0111\rangle + |1101\rangle + |1011\rangle \\
&\quad - |0010\rangle - |0100\rangle - |1000\rangle - |0001\rangle), \quad (33)\\
|\varphi_1\rangle_3 &= \frac{1}{\sqrt{8}}(-|0000\rangle + |1111\rangle - |0110\rangle - |1100\rangle \\
&\quad + |1010\rangle + |0011\rangle - |0101\rangle + |1001\rangle), \quad (34)\\
|\varphi_2\rangle_0 &= \frac{1}{\sqrt{8}}(-|0000\rangle + |1111\rangle + |0110\rangle - |1100\rangle \\
&\quad - |1010\rangle + |0011\rangle + |0101\rangle - |1001\rangle, \quad (35)\\
|\varphi_2\rangle_1 &= \frac{1}{\sqrt{8}}(-|1110\rangle + |0111\rangle - |1101\rangle + |1011\rangle \\
&\quad + |0010\rangle - |0100\rangle - |1000\rangle + |0001\rangle), \quad (36)\\
|\varphi_2\rangle_2 &= \frac{1}{\sqrt{8}}(|1110\rangle - |0111\rangle + |1101\rangle + |1011\rangle \\
&\quad + |0010\rangle + |0100\rangle - |1000\rangle + |0001\rangle), \quad (37)\\
|\varphi_2\rangle_3 &= \frac{1}{\sqrt{8}}(|0000\rangle + |1111\rangle + |0110\rangle - |1100\rangle \\
&\quad + |1010\rangle - |0011\rangle + |0101\rangle + |1001\rangle), \quad (38)\\
|\varphi_3\rangle_0 &= \frac{1}{\sqrt{8}}(-|0000\rangle + |1111\rangle - |0110\rangle + |1100\rangle \\
&\quad - |1010\rangle - |0011\rangle + |0101\rangle + |1001\rangle), \quad (39)\\
|\varphi_3\rangle_1 &= \frac{1}{\sqrt{8}}(-|1110\rangle - |0111\rangle + |1101\rangle + |1011\rangle \\
&\quad - |0010\rangle - |0100\rangle + |1000\rangle + |0001\rangle), \quad (40)\\
|\varphi_3\rangle_2 &= \frac{1}{\sqrt{8}}(|1110\rangle + |0111\rangle - |1101\rangle + |1011\rangle \\
&\quad - |0010\rangle + |0100\rangle + |1000\rangle + |0001\rangle), \quad (41)\\
|\varphi_3\rangle_3 &= \frac{1}{\sqrt{8}}(|0000\rangle + |1111\rangle - |0110\rangle + |1100\rangle \\
&\quad + |1010\rangle + |0011\rangle + |0101\rangle - |1001\rangle), \quad (42)\\
|\varphi_4\rangle_0 &= \frac{1}{\sqrt{8}}(|0000\rangle + |1111\rangle + |0110\rangle + |1100\rangle \\
&\quad - |1010\rangle + |0011\rangle - |0101\rangle + |1001\rangle), \quad (43)\\
|\varphi_4\rangle_1 &= \frac{1}{\sqrt{8}}(-|1110\rangle + |0111\rangle + |1101\rangle + |1011\rangle \\
&\quad + |0010\rangle + |0100\rangle + |1000\rangle - |0001\rangle), \quad (44)\\
|\varphi_4\rangle_2 &= \frac{1}{\sqrt{8}}(|1110\rangle - |0111\rangle - |1101\rangle + |1011\rangle \\
&\quad + |0010\rangle - |0100\rangle + |1000\rangle - |0001\rangle), \quad (45)\\
|\varphi_4\rangle_3 &= \frac{1}{\sqrt{8}}(-|0000\rangle + |1111\rangle + |0110\rangle + |1100\rangle \\
&\quad + |1010\rangle - |0011\rangle - |0101\rangle - |1001\rangle). \quad (46)
\end{aligned}$$

It is straightforward to check that ${}_{\nu}\langle\varphi_{\mu}|\varphi_{\mu'}\rangle_{\upsilon'} = \delta_{\mu\mu'}\delta_{\nu\nu'}$ and $\sum_{\mu\nu}|\varphi_{\mu}\rangle_{\nu\nu}\langle\varphi_{\mu}| = 1$. This implies that above sixteen states form a orthonormal and completeness Hilbert space of the four-qubit system. In fact, they build a new type representation of the four-qubit system, i.e., a genuine entangled-state representation. In this representation an arbitrary state of the four-qubit system can be expressed in terms of the basis of the representation. In order to see this, we consider the typical four-qubit entangled states [15, 16, 17, 18]: the GHZ state $|\mathtt{GHZ_4}\rangle$, the W state $|W_4\rangle$, the cluster state $|\mathtt{CL_4}\rangle$, and the symmetric Dicke state $|D_4\rangle$. They have the following expressions, respectively,

$$\begin{aligned}
|\mathtt{GHZ_4}\rangle &= \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle), \quad (47)\\
|W_4\rangle &= \frac{1}{2}(|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle), \quad (48)\\
|\mathtt{CL_4}\rangle &= \frac{1}{2}(|0000\rangle + |0110\rangle + |1001\rangle - |1111\rangle), \quad (49)\\
|D_4\rangle &= \frac{1}{\sqrt{6}}(|0011\rangle + |0101\rangle + |1001\rangle \\
&\quad + |1100\rangle + |0110\rangle + |1010\rangle). \quad (50)
\end{aligned}$$

Making use of the basis of the genuine entangled-state representation given by Eqs. (31-46), we can find that above four typical four-qubit entangled states can be expressed as follows:

$$\begin{aligned}
|\mathtt{GHZ_4}\rangle &= \frac{1}{2}\left(|\varphi_1\rangle_0 + |\varphi_3\rangle_3 + |\varphi_2\rangle_3 + |\varphi_4\rangle_0\right), \quad (51)\\
|W_4\rangle &= \frac{1}{\sqrt{8}}(|\varphi_3\rangle_2 + |\varphi_2\rangle_2 - |\varphi_1\rangle_1 - 2|\varphi_1\rangle_2 \\
&\quad + |\varphi_4\rangle_1), \quad (52)\\
|\mathtt{CL_4}\rangle &= \frac{1}{\sqrt{8}}(|\varphi_2\rangle_3 + |\varphi_4\rangle_0 - |\varphi_1\rangle_0 - |\varphi_3\rangle_0 \\
&\quad - |\varphi_1\rangle_3 - |\varphi_3\rangle_3 - |\varphi_4\rangle_3 - |\varphi_2\rangle_0), \quad (53)\\
|D_4\rangle &= \frac{1}{2\sqrt{3}}(2|\varphi_2\rangle_3 - |\varphi_4\rangle_3 + |\varphi_2\rangle_0 + |\varphi_1\rangle_3 \\
&\quad - |\varphi_3\rangle_0 - 2|\varphi_1\rangle_0). \quad (54)
\end{aligned}$$

Finally, we consider the influence of the imperfection of photon detections in the present scheme. An ideal photon detection with quantum efficiency $\eta = 1$ can be described by the positive operator-valued measure (POVM) of each detector $\{\Pi_0 = |0\rangle\langle 0|, \Pi_1 = I - \Pi_0\}$. In the realistic case, an incoming photon can not be detected with the probability success 1. If the quantum efficiency of the photodetector is η, the POVM is given by [12, 13, 14]

$$\Pi_0(\eta) = \sum_{i=0}(1-\eta)^i|i\rangle\langle i|, \quad \Pi_1(\eta) = I - \Pi_0(\eta), \quad (55)$$

from which it is straightforward to find that the inefficiency of photodetectors does not affect the quality of

the generated entangled states, but it decreases the success probability. In our scheme the success probability of the state to obtain the GESs $|\overline{\chi'}\rangle$ and $|\overline{\chi''}\rangle$ are η^2 when the quantum efficiency of each photodetector is η.

IV. CONCLUDING REMARKS

In conclusion, we have proposed a theoretical scheme to generate genuine entangled states of four atomic qubits in separated optical cavities using the atom-light interaction under the condition of the large detuning and single photon detections. We have shown that GESs of four atomic qubits can be produced deterministically. Starting with one prepared GES we have found the sixteen orthonormal and independent GESs. We have shown that these sixteen GESs build a new type of representation the four-qubit system, the genuine entangled-state representation. This representation provides us with new interesting insight into better understanding multipartite entanglement. It have indicated that the GHZ state and W state, the cluster state, and the symmetric Dicke state for a four-qubit system can be explicitly expressed in terms of the sixteen GESs. We have considered the influence of the imperfection of photodetectors in the present scheme, and indicated that the inefficiency of photodetectors does not affect the quality of the generated entangled states, but it decreases the success probability. It is believed that the GESs created in the present scheme provide new entanglement sources to realize quantum information processing.

Acknowledgments

This work was supported by the National Fundamental Research Program Grant No. 2007CB925204, the National Natural Science Foundation under Grant Nos. 10775048 and 10325523, and the Education Committee of Hunan Province under Grant No. 08W012.

[1] Shor P W 1995 *Phys. Rev.* A **52** R2493
[2] Hillery M, Bužek V, and Berthiaume A 1999 *Phys. Rev.* A **59** 1829
[3] Murao M, Jonathan D, Plenio M B, and Vedral V 1999 *Phys. Rev.* A **59** 156
[4] Raussendorf R and Briegel H J 2001 *Phys. Rev. Lett* **86** 5188
[5] de Oliveira T R, Rigolin G, and de Oliveira M C 2006 *Phys. Rev.* A **73** 010305
[6] Yeo Y and Chua W K 2006 *Phys. Rev. Lett.* **96** 060502
[7] Chen P X, Zhu S Y, and Guo G C 2006 *Phys. Rev.* A **74** 032324
[8] Z. X. Man, Y. J. Xia, and N. B. An 2007 *Phys. Rev.* A **75** 052306
[9] Duan L M, Cirac J I, Zoller P, and Polzik E S 2000 *Phys. Rev Lett.* **85** 5643
[10] Huang Y P and Moore M G 2008 *Phys. Rev.* A **77** 032349
[11] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245
[12] Kok P, Munro W J, Nemoto K, Ralph T C, Dowling J P, and Milburn G J 2007 *Rev. Mod. Phys.* **79** 135
[13] Kuang L M, Chen Z B, Pan J W 2007 *Phys. Rev.* A **76** 052324
[14] Olivares S, Paris M G A, and Bonifacio R 2003 *Phys. Rev.* A **67** 032314
[15] Greenberger D M, Horne M A, and Zeilinger A 1989 in *Bell's Theorem, Quantum Theory, and Conceptions of the Universe* edited by M. Kafatos (Dordrecht: Kluwer)
[16] Dür W, Vidal G, and Cirac J I 2000 *Phys. Rev.* A **62** 062314
[17] Briegel H J and Raussendorf R 2001 *Phys. Rev. Lett.* **86** 910
[18] Dicke R H 1954 *Phys. Rev.* **93** 99; G. Tóth 2007 *J. Opt. Soc. Am.* B **24** 275